EXERCICE 7-3

1) <u>Bilan</u>

* Pesanteur en G_T : $\vec{P_T}\begin{vmatrix} +mg\sin\alpha \\ -mg\cos\alpha \\ 0 \end{vmatrix}_{b_0}$ $\qquad \vec{M}_{P_T}(G_T) = \vec{0}$

* Pesanteur en C : $\vec{P_D}\begin{vmatrix} +Mg\sin\alpha \\ -Mg\cos\alpha \\ 0 \end{vmatrix}_{b_0}$ $\qquad \vec{M}_{P_D}(C) = \vec{0}$

* Action en A : $\vec{R_A}\begin{vmatrix} T_A \\ N_A \\ 0 \end{vmatrix}_{b_0}$ $\qquad \vec{M_A}(A) = \vec{0}$

* Action en I : $\vec{R_I}\begin{vmatrix} T_I \\ N_I \\ 0 \end{vmatrix}_{b_0}$ $\qquad \vec{M_I}(I) = \vec{0}$

2) <u>Résultante</u> $\vec{P} = \vec{P_T} + \vec{P_D}$ voir schéma

3) On suppose être à la limite du glissement au point I, l'action $\vec{R_I}$ se trouve sur le cône de frottement. Pour que le jouet soumis à $\vec{P}, \vec{R_I}, \vec{R_A}$ soit en équilibre il faut que les trois directions de ces forces soient concourantes. d'où on déduit $\Delta\vec{R_A}$, et donc φ_A

4) l'équilibre est impossible car $\varphi_A > 45°$ et donc $f_A > 1$.

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5) mouvement de (T) = translation. $\Rightarrow \vec{\Omega}_T = \vec{0}$

6) $\mathbb{I}(A,T,b_2)\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{mL^2}{3} & 0 \\ 0 & 0 & \frac{mL^2}{3} \end{bmatrix}$ $\qquad \vec{AG_T} = \begin{vmatrix} a = L/2 \\ b = 0 \\ c = 0 \end{vmatrix}_{b_2}$

$A_A = A_{G_T} + m(b^2+c^2) \Rightarrow A_{G_T} = A_A$

$B_A = B_{G_T} + m(a^2+c^2) \Rightarrow B_{G_T} = B_A - m\frac{L^2}{4} = \frac{mL^2}{12}$

$C_A = C_{G_T} + m(a^2+b^2) \Rightarrow C_{G_T} = C_A - \frac{mL^2}{4} = \frac{mL^2}{12}$

$\Rightarrow \mathbb{I}(G_T,T,b_2)\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{mL^2}{12} & 0 \\ 0 & 0 & \frac{mL^2}{12} \end{bmatrix}$

$D_A = D_{G_T} + mbc \Rightarrow D_{G_T} = D_A$

$E_A = E_{G_T} + mac \Rightarrow E_{G_T} = E_A$

$F_A = F_{G_T} + mab \Rightarrow F_{G_T} = F_A$

7) * $\vec{R_T}(V) = m\,\vec{V}(G_T)$

$$\vec{OG_T} = \vec{OC} + \vec{CG_T} = x\,\vec{x_0} + R\,\vec{y_0} - \frac{L}{2}\vec{x_2}$$

$$\vec{V}(G_T) = \dot{x}\,\vec{x_0}$$

* $\vec{\sigma_T}(G) = \mathbb{I}(G_T, T, b_2) \cdot \vec{\Omega_T} = \vec{0}$

$$\vec{\sigma_T}(C) = \vec{\sigma_T}(G_T) + \vec{R_T}(V) \wedge \vec{G_T C}$$

$$= \vec{0} + m\,\dot{x}\,\vec{x_0} \wedge \frac{L}{2}\vec{x_2}$$

$$= m\frac{L}{2}\dot{x}\sin\beta\,\vec{z}$$

$$\Rightarrow \begin{cases} \vec{R_T}(V) = m\,\dot{x}\,\vec{x_0} \\ \vec{\sigma_T}(C) = m\frac{L}{2}\dot{x}\sin\beta\,\vec{z} \end{cases}$$

8) * $\vec{R_T}(\Gamma) = m\,\vec{\gamma}(G_T) = m\,\ddot{x}\,\vec{x_0}$

* $$\vec{\delta_T}(C) = \frac{d\,\vec{\sigma_T}(C)}{dt} + m\,\vec{V}(G_T) \wedge \vec{V}(C)$$

$$= m\frac{L}{2}\ddot{x}\sin\beta\,\vec{z} + \underbrace{m\,\dot{x}\,\vec{x_0} \wedge \dot{x}\,\vec{x_0}}_{\vec{0}}$$

$$\Rightarrow \begin{cases} \vec{R_T}(\Gamma) = m\,\ddot{x}\,\vec{x_0} \\ _T(C) = m\frac{L}{2}\ddot{x}\sin\beta\,\vec{z} \end{cases}$$

9) $$E_{CT} = \frac{1}{2}m\,\vec{V}(G_T)^2 + \frac{1}{2}\vec{\Omega_T} \cdot \vec{\sigma}(G_T)$$

$$= \frac{1}{2}m\,\dot{x}^2$$

10) Non calculable pour cause de glissement en A

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11) Disque Translation + Rotation $\Rightarrow \vec{\Omega_D} = \dot{\theta}\,\vec{z}$

12) $$\vec{U_{DP}} = \vec{V}(I \in D) - \underbrace{\vec{V}(I \in P)}_{\vec{0}}$$

$$\vec{V}(I \in D) = \vec{V}(C) + \vec{\Omega_D} \wedge \vec{CI} = \dot{x}\,\vec{x_0} + \dot{\theta}\,\vec{z} \wedge -R\,\vec{y_0} = (\dot{x} + R\dot{\theta})\,\vec{x_0}$$

CRSG : $\dot{x} + R\dot{\theta} = 0$

13) * $\vec{R_D}(V) = M\,\vec{V}(C) = M\dot{x}\,\vec{x_0}$

* $\vec{\sigma_D}(C) = \mathbb{I}(C, D, b_1)\,\vec{\Omega_D}$

$$= \begin{bmatrix} \frac{MR^2}{4} & 0 & 0 \\ 0 & \frac{MR^2}{4} & 0 \\ 0 & 0 & \frac{MR^2}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \Rightarrow \begin{cases} \vec{R_D}(V) = M\dot{x}\,\vec{x_0} \\ \vec{\sigma_D}(C) = \frac{MR^2}{2}\dot{\theta}\,\vec{z} \end{cases}$$

$$= \frac{MR^2}{2}\dot{\theta}\,\vec{z}$$

14) * $\overrightarrow{R_D}(\gamma) = M\ddot{x}\,\overrightarrow{x_0}$

* $\overrightarrow{\delta_D}(C) = \frac{d}{dt}\overrightarrow{\sigma_D}(C) = \frac{MR^2}{2}\ddot{\theta}\,\vec{z}$

15) $E_C = \frac{1}{2}M\,\overrightarrow{V(C)}^2 + \frac{1}{2}\overrightarrow{\Omega_D}\cdot\overrightarrow{\sigma_D(C)}$

$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}\frac{MR^2}{2}\dot{\theta}^2$

$\to h_C = \overrightarrow{OC}\cdot\vec{y}$
$= x\,\overrightarrow{x_0}\cdot\vec{y} + R\,\overrightarrow{y_0}\cdot\vec{y}$
$= -x\sin\alpha + cst.$

16) $E_p = Mg\,h_C + cst = -Mg\,x\sin\alpha + cst.$
sous l'hypothèse qu'il y a roulement sans glissement en I !

17) $(M+m)\overrightarrow{CG} = m\overrightarrow{CG_T} + M\overrightarrow{CC}$

sur $(\overrightarrow{x_2})$ $\quad M+m\ \overrightarrow{CG}\cdot\overrightarrow{x_2} = -m\frac{L}{2}\overrightarrow{x_2} + \vec{0}$

$\Rightarrow \overrightarrow{CG} = \frac{-mL}{2(M+m)}\overrightarrow{x_2} = -\frac{L}{8}\overrightarrow{x_2}$

18) * $\overrightarrow{R_J}(\gamma) = \overrightarrow{R_T}(\gamma) + \overrightarrow{R_D}(\gamma) = (M+m)\ddot{x}\,\overrightarrow{x_0}$

* $\overrightarrow{\delta_J}(C) = \overrightarrow{\delta_T}(C) + \overrightarrow{\delta_D}(C) = m\frac{L}{2}\ddot{x}\sin\beta\,\overrightarrow{z_0} + \frac{MR^2}{2}\ddot{\theta}\,\vec{z}$

$= \left(m\frac{L}{2}\ddot{x}\sin\beta + \frac{MR^2}{2}\ddot{\theta}\right)\vec{z}$

19) * $E_{CJ} = E_{CT} + E_{CD} = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{4}MR^2\dot{\theta}^2$

20) * $E_{pJ} = E_{pT} + E_D$ non calculable car glissement en A.

21) - NON $\Rightarrow$ glissement au point A.

22)
* $\overrightarrow{\mathcal{M}_{P_T}}(C) = \overrightarrow{\mathcal{M}_{P_T}}(G_T) + \overrightarrow{P_T}\wedge\overrightarrow{G_TC}$ $\qquad \overrightarrow{G_TC} = \frac{L}{2}\overrightarrow{x_2} = \frac{L}{2}(\cos\beta\,\overrightarrow{x_0} + \sin\beta\,\overrightarrow{y_0})$

$= \begin{vmatrix}0\\0\\0\end{vmatrix} + \begin{vmatrix}+mg\sin\alpha\\-mg\cos\alpha\\0\end{vmatrix}\wedge\begin{vmatrix}\frac{L}{2}\cos\beta\\\frac{L}{2}\sin\beta\\0\end{vmatrix}$

$= +mg\frac{L}{2}(\sin\alpha\sin\beta + \cos\alpha\cos\beta)\,\vec{z} = mg\frac{L}{2}\cos(\alpha-\beta)\,\vec{z}$

* $\overrightarrow{\mathcal{M}_{A}}(C) = \overrightarrow{\mathcal{M}_A}(A) + \overrightarrow{R_A}\wedge\overrightarrow{AC}$ $\qquad \overrightarrow{AC} = L\overrightarrow{x_2}$

$= \begin{vmatrix}0\\0\\0\end{vmatrix} + \begin{vmatrix}T_A\\N_A\\0\end{vmatrix}\wedge\begin{vmatrix}L\cos\beta\\L\sin\beta\\0\end{vmatrix}$

$= L(T_A\sin\beta - N_A\cos\beta)\,\vec{z}$

$* \ \vec{M_I}(C) = \vec{M_I}(I) + \vec{R_I} \wedge \vec{IC} \qquad \vec{IC} = R\vec{y_0}$

$= \vec{0} + (T_I \vec{x_0} + N_I \vec{y_0}) \wedge R\vec{y_0} = T_I . R \vec{z}$

$$\Rightarrow \vec{R_{Fext}} = \left| \begin{array}{l} +(M+m) g \sin \alpha + T_A + T_I \\ -(M+m) g \cos \alpha + N_A + N_I \\ 0 \end{array} \right._{b_0}$$

$$\vec{M_{Fext}}(C) = \Big[+ mg\frac{L}{2} \cos(\alpha - \beta) + L(T_A \sin\beta - N_A \cos\beta) + T_I R \Big] \vec{z}$$

2.3) $\vec{R_{Fext}} = \vec{R_J}(\delta)$

$\vec{M_{Fext}}(C) = \vec{\delta_J}(C)$

$$\Rightarrow \left| \begin{array}{l} +(M+m) g \sin \alpha + T_A + T_I = (M+m) \ddot{x} \\ -(M+m) g \cos \alpha + N_A + N_I = 0 \\ -mg\frac{1}{2} \times \cos(\alpha - \beta) + L(T_A \sin\beta - N_A \cos\beta) + T_I . R = \frac{ML}{2} \ddot{x} \sin\beta + \frac{MR^2}{2} \ddot{\theta} \end{array} \right.$$