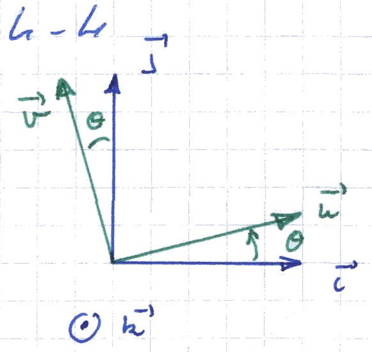
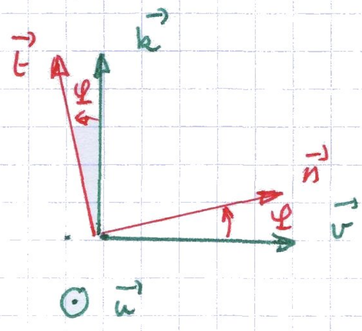
4 - 4





$\vec{\Omega}_{01} = \dot{\theta}\,\vec{k}$ $\qquad$ $\vec{\Omega}_{12} = \dot{\varphi}\,\vec{u}$ $\qquad$ $\vec{\Omega}_{02} = \dot{\theta}\,\vec{k} + \dot{\varphi}\,\vec{u}$

a) $\begin{cases} \vec{n} = \cos\varphi\,\vec{v} + \sin\varphi\,\vec{k} \\ \vec{t} = -\sin\varphi\,\vec{v} + \cos\varphi\,\vec{k} \end{cases}$

b) $\begin{cases} \vec{u} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} \\ \vec{v} = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j} \end{cases}$

$\vec{n} = \cos\varphi\,(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}) + \sin\varphi\,\vec{k}$

$\vec{n} = -\cos\varphi\sin\theta\,\vec{i} + \cos\varphi\cos\theta\,\vec{j} + \sin\varphi\,\vec{k}$

$\vec{t} = -\sin\varphi\,(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}) + \cos\varphi\,\vec{k}$

$t = \sin\varphi\sin\theta\,\vec{i} - \sin\varphi\cos\theta\,\vec{j} + \cos\varphi\,\vec{k}$

c) $\vec{OM} = R\,\vec{n}$

$= -R\cos\varphi\cos\theta\,\vec{i} + R\cos\varphi\cos\theta\,\vec{j} + R\sin\varphi\,\vec{k}$

$\vec{OM}\,(-R\cos\varphi\cos\theta,\ R\cos\varphi\cos\theta,\ R\sin\varphi)$

d) $\vec{V}_0(M) = \dfrac{d_0\vec{OM}}{dt} = \dfrac{d_0 R\vec{n}}{dt} = R\,\dfrac{d_0\vec{n}}{dt} = R(\vec{\Omega}_{02} \wedge \vec{n})$

$= R(\dot{\theta}\vec{k} + \dot{\varphi}\vec{u}) \wedge \vec{n} = R\dot{\theta}\,\vec{k}\wedge\vec{n} + R\dot{\varphi}\,\vec{u}\wedge\vec{n}$

$= R\dot{\theta}\,\sin(-\frac{\pi}{2}+\varphi)\,\vec{u} + R\dot{\varphi}\,\vec{t}$

$= -R\dot{\theta}\cos\varphi\,\vec{u} + R\dot{\varphi}\,\vec{t}$

$\vec{\gamma}_0(M) = \dfrac{d_0^2\vec{OM}}{dt} = \dfrac{d_0 V(M)}{dt} =$

$$= -R\ddot{\theta}\cos\varphi\,\vec{u} + R\dot{\theta}\dot{\varphi}\sin\varphi\,\vec{u} - R\dot{\theta}\cos\varphi\,\frac{d_0\vec{u}}{dt}$$

$$+ R\ddot{\varphi}\,\vec{t} + R\dot{\varphi}\,\frac{d_0\vec{t}}{dt}$$

$\frac{d_0\vec{u}}{dt}$: $\vec{\Omega}_{01}\wedge\vec{u} = \dot{\theta}\vec{k}\wedge\vec{u} = \dot{\theta}\vec{v}$

$=$ $\quad$ $\frac{d_0\vec{t}}{dt}$: $\vec{\Omega}_{02}\wedge\vec{t} = (\dot{\theta}\vec{k} + \dot{\varphi}\vec{u})\wedge\vec{t} = \dot{\theta}\vec{k}\wedge\vec{t} + \dot{\varphi}\vec{u}\wedge\vec{t}$

$$= \dot{\theta}\sin\varphi\,\vec{u} + \dot{\varphi}(-\vec{n})$$

$$= (-R\ddot{\theta}\cos\varphi + 2R\dot{\theta}\dot{\varphi}\sin\varphi)\,\vec{u} - R\dot{\theta}^2\cos\varphi\,\vec{v} + R\ddot{\varphi}\,\vec{t} - R\dot{\varphi}^2\,\vec{n}$$

e) $\vec{V}_1(M) = \frac{d_1\vec{OM}}{dt}$ (avec b_1 supposée fixe ! $\Leftrightarrow \theta = cste$)

$$= \frac{d_1 R\vec{n}}{dt} = R\frac{d_1\vec{n}}{dt} = R\,\vec{\Omega}_{12}\wedge\vec{n} = R\dot{\varphi}\vec{u}\wedge\vec{n} = R\dot{\varphi}\,\vec{t}$$

$$\vec{\gamma}_1(M) = \frac{d_1^2\vec{OM}}{dt^2} = \frac{d_1\vec{V}_1(M)}{dt} = R\ddot{\varphi}\,\vec{t} + R\dot{\varphi}\frac{d_1\vec{t}}{dt}$$

$$= R\ddot{\varphi}\,\vec{t} + R\dot{\varphi}\,\vec{\Omega}_{12}\wedge\vec{t} = R\ddot{\varphi}\,\vec{t} - R\dot{\varphi}^2\,\vec{n}$$