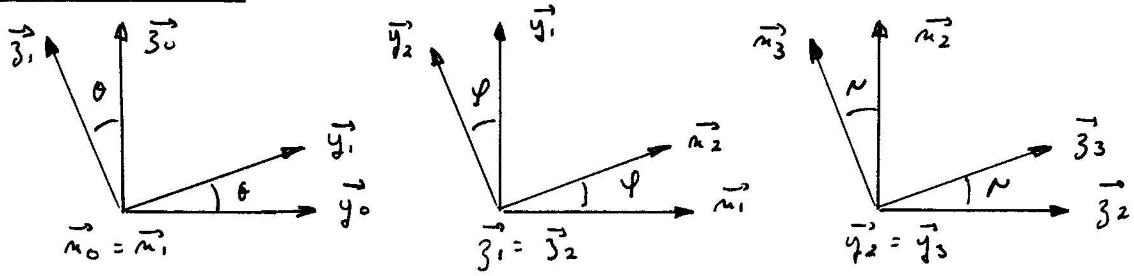
EXERCICE 1-6

a)



b)

$$\vec{x_1} = \vec{x_0}$$
$$\vec{y_1} = \cos\theta\, \vec{y_0} + \sin\theta\, \vec{z_0}$$
$$\vec{z_1} = -\sin\theta\, \vec{y_0} + \cos\theta\, \vec{z_0}$$

c)

$$\vec{x_1} = \cos\varphi\, \vec{x_2} - \sin\varphi\, \vec{y_2}$$
$$\vec{y_1} = \sin\varphi\, \vec{x_2} + \cos\varphi\, \vec{y_2}$$
$$\vec{z_1} = \vec{z_2}$$

d)

$$\vec{y_1} \cdot \vec{x_2} = \cos\left(\frac{3\pi}{2} + \varphi\right) = \sin\varphi$$
$$\vec{x_1} \cdot \vec{y_2} = \cos\left(\frac{\pi}{2} + \varphi\right) = -\sin\varphi$$
$$\vec{x_2} \cdot \vec{z_3} = \cos\left(\frac{3\pi}{2} + \psi\right) = \sin\psi$$
$$\vec{z_2} \cdot \vec{x_3} = \cos\left(\frac{\pi}{2} + \psi\right) = -\sin\psi$$

e)

$$\vec{z_2} \wedge \vec{z_3} = \sin\psi\, \vec{y_2}$$
$$\vec{y_2} \wedge \vec{z_3} = \vec{y_3} \wedge \vec{z_3} = \vec{x_3}$$
$$\vec{z_2} \wedge \vec{x_3} = \sin\left(\frac{\pi}{2} + \psi\right) \vec{y_2} = \cos\psi\, \vec{y_2}$$
$$\vec{y_2} \wedge \vec{x_1} = \sin\left(-\varphi + \frac{3\pi}{2}\right) \vec{z_1} = -\cos\varphi\, \vec{z_1}$$

f)

$$\vec{x_1} \wedge \vec{z_3} = (\cos\varphi\, \vec{x_2} - \sin\varphi\, \vec{y_2}) \wedge \vec{z_3} = \cos\varphi\, (\vec{x_2} \wedge \vec{z_3}) - \sin\varphi\, (\vec{y_2} \wedge \vec{z_3})$$
$$= \cos\varphi \sin\left(\frac{3\pi}{2} + \psi\right) \vec{y_2} - \sin\varphi\, \vec{x_3} = -\cos\varphi \cos\psi\, \vec{y_2} - \sin\varphi\, \vec{x_3}$$

ou $\vec{x_1} \wedge \vec{z_3} = \vec{x_1} \wedge (\cos\psi\, \vec{z_2} + \sin\psi\, \vec{x_2}) = \cos\psi\, \vec{x_1} \wedge \vec{z_2} + \sin\psi\, \vec{x_1} \wedge \vec{x_2}$

$(\vec{z_1} = \vec{z_2}) \Rightarrow$ $= -\cos\psi\, \vec{y_1} + \sin\psi \sin\varphi\, \vec{z_1} = -\cos\psi(\cos\varphi\, \vec{y_2} + \sin\varphi\, \vec{x_2}) + \sin\psi \sin\varphi\, \vec{z_2} =$

$$= -\cos\psi \sin\varphi\, \vec{x_2} - \cos\psi \cos\varphi\, \vec{y_2} + \sin\psi \sin\varphi\, \vec{z_2}$$
$$= -\sin\varphi\, (\cos\psi\, \vec{x_2} - \sin\psi\, \vec{z_2}) - \cos\psi \cos\varphi\, \vec{y_2} = -\sin\varphi\, \vec{x_3} - \cos\varphi \cos\psi\, \vec{y_2}$$

$$\vec{y_1} \wedge \vec{y_3} = \vec{y_1} \wedge \vec{y_2} = \sin\varphi\, \vec{z_1}$$
$$\vec{z_2} \wedge \vec{z_1} = \vec{0}$$